Structure and Evolution of Social Networks

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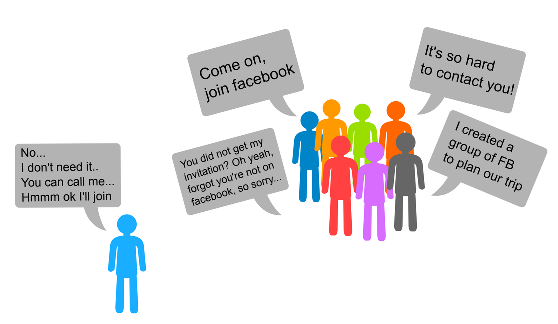
Investigation plan

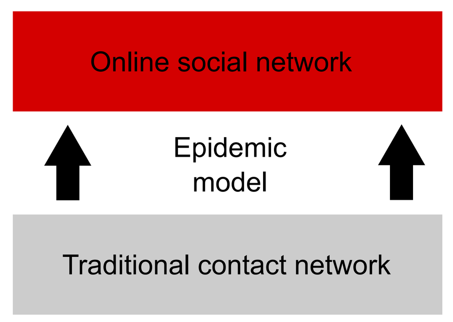
Emergence of Online Social Networks

Infection based multiplex model

Social contaigon

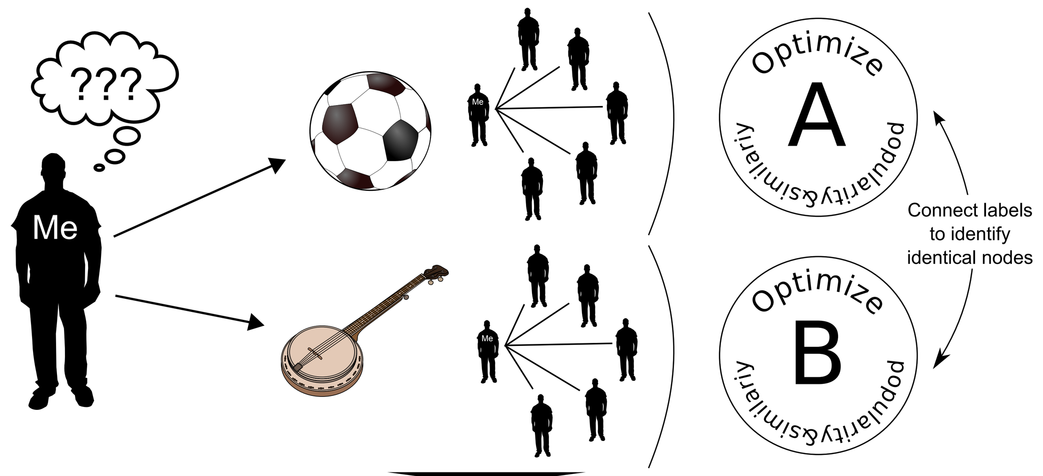


Two layer structure



Social influence drives OSN subscriptions and intrinsically leads to multiplex structure.

Multidimensional popularity/similarity model

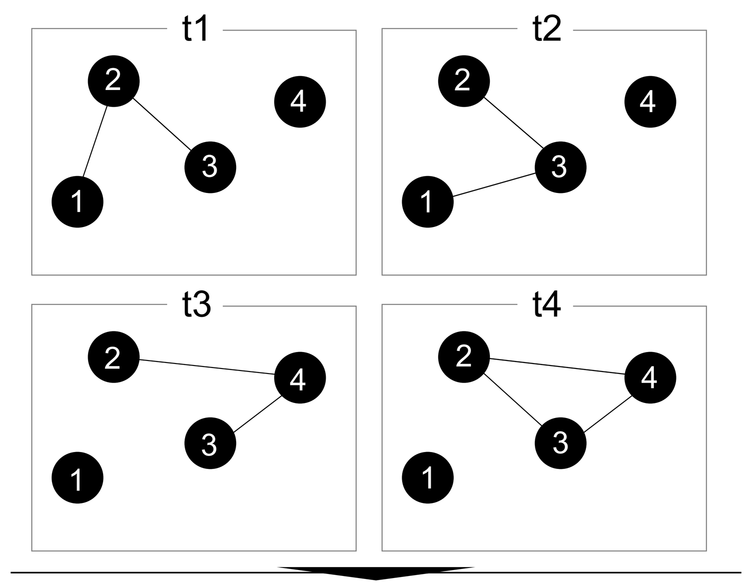


Individuals optimize popularity and similarity in multiple popularity/similarity spaces.

Papadopoulos, Kitsak, Serrano, Boguñá, Krioukov (Popularity versus similarity in growing networks, 2012)

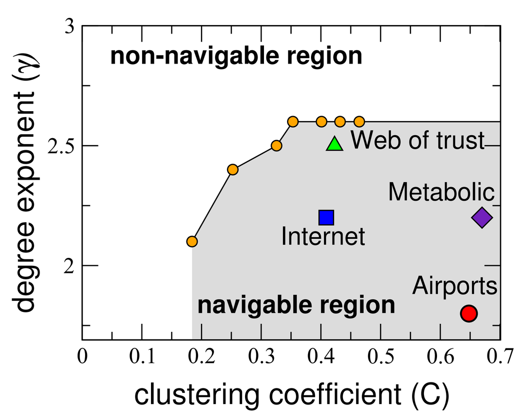
Temporal networks and navigability

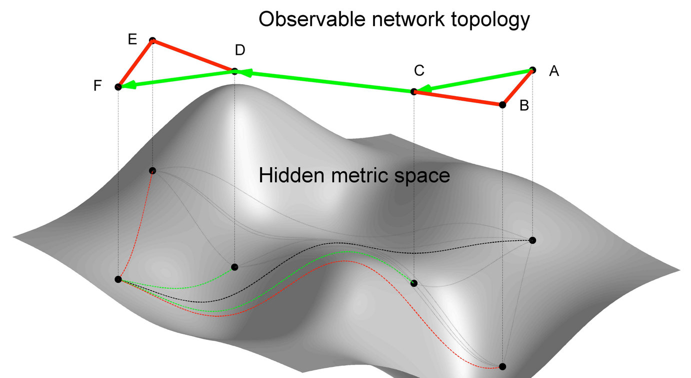
Links can be active or inactive



$1 \rightarrow 4$ ✓ $4 \rightarrow 1$ ✗

Generalization of phase diagram and metric





Boguñá et al (Navigability of complex networks, 2008)

Investigate effects of link activity on navigability and add temporal dimension to phase diagram.

Non-Markovian dynamics on complex Networks

Markovian dynamics

- Well studied on complex networks
- For example epidemic spreading
- Good approximation for many problems
- May not be appropriate for dynamics of social interactions

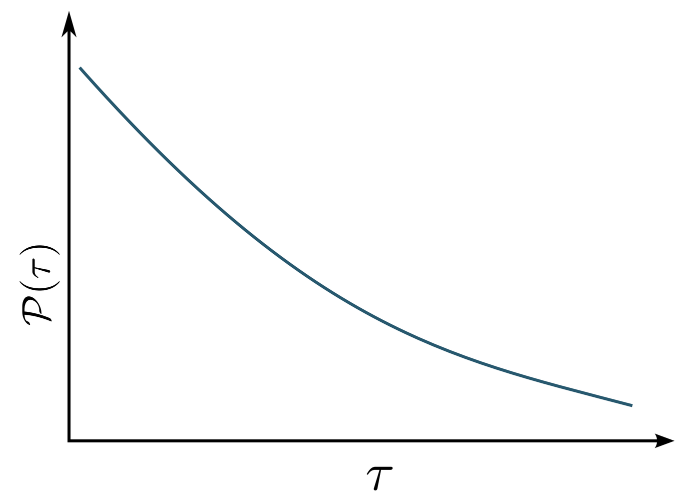
Non-Markovian dynamics

- Barely studied on complex networks
- Very important for social interactions
- Interesting: integro-differential equations of the form

$$\frac{\mathrm{d}\varrho}{\mathrm{d}t} = \int_0^t \mathrm{d}t' \, \psi(t-t') F(\varrho, t')$$

in mean-field approximation

Non-Markovian probability



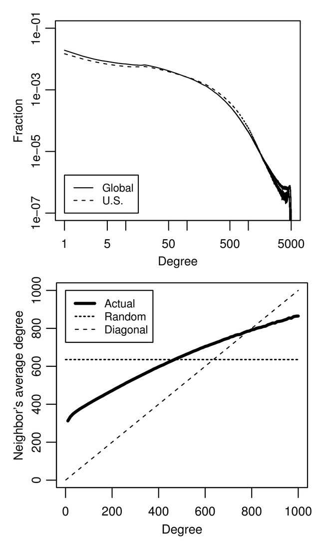
Investigation of non-Markovian dynamics is essential to understand social interactions.

Probability depends on time since last event occured.

Ongoing work

Empiric data

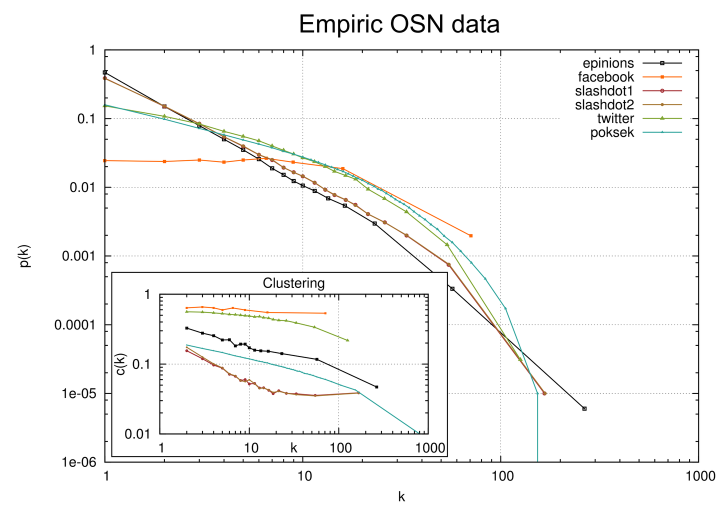
Facebook



Degree distribution, clustering spectrum, and degree correlations for facebook graph. Taken from Ugander, et al, The anatomy of the facebook social graph, 2011.

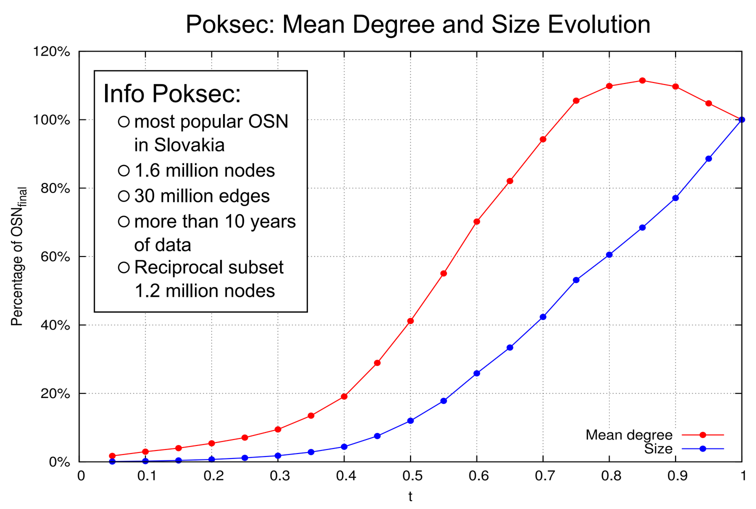
Analysis of graph of ~720 million nodes.

Degree and clustering of empiric OSNs



High clustering and heavy tailed degree distribution.

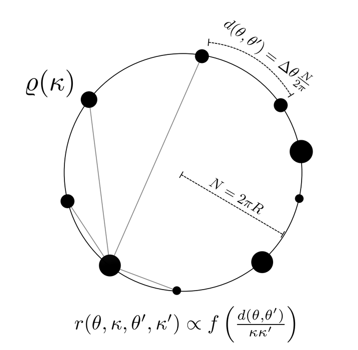
Evolution of Poksec (Slovakian OSN)



Evolution of Poksec can be compared with models.

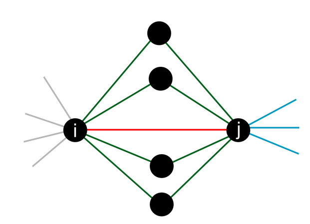
Infection model

S1 - Model

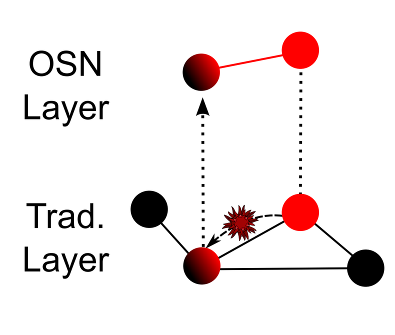


Add weights

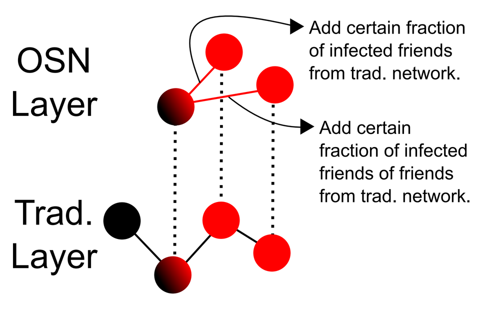
$$\omega(i,j) \propto \frac{N(\blacksquare)}{N(\blacksquare)+N(\blacksquare)}$$



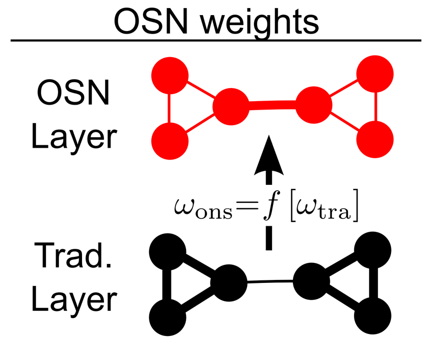
Infect



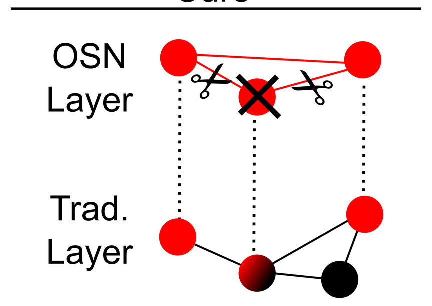
Connections



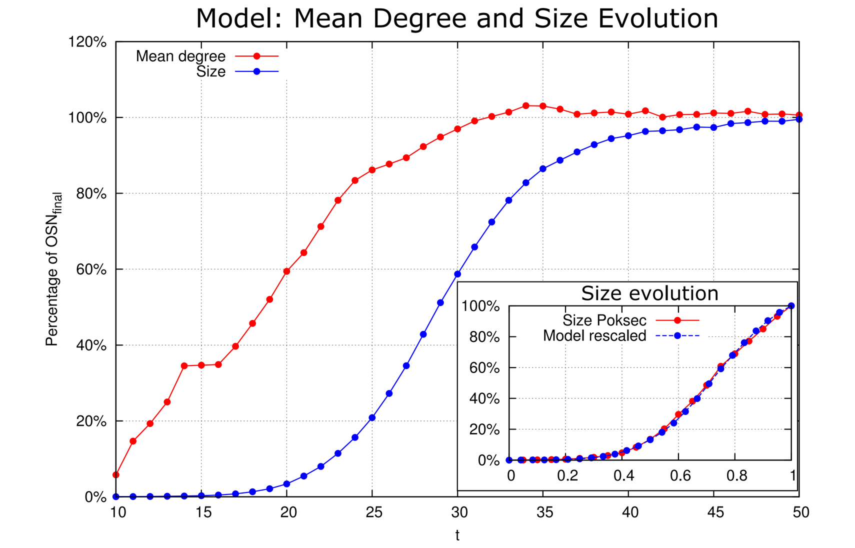
OSN weights



Cure



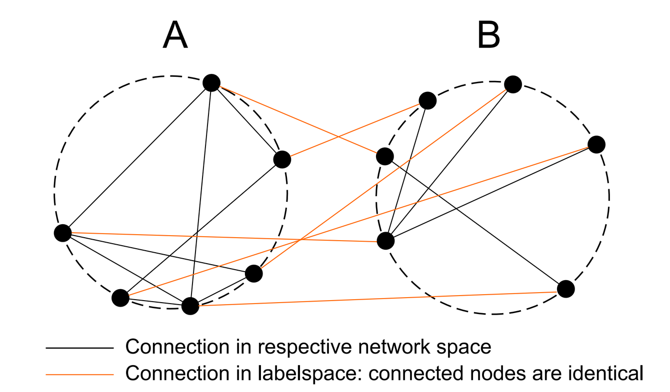
Size and mean degree evolution



Size evolution of infection model coincides well with Poksec network.

Multidimensional similarity/popularity model

Multidimensional layout



Two networks created according to similarity/popularity model independently. Then, we make interconnections to label same vertices.

- Interconnections

$\varrho(t_a, t_b)$ joint probability for nodes a,b to have t_a, t_b.

- Expected degree

$$\bar{k}(t_a, t_b) = C_a \left[\frac{T}{t_a}\right]^{z_a} + C_b \left[\frac{T}{t_b}\right]^{z_b}$$

Collapsed aggregated network

- Degree distribution 1D

$$p(k) = \frac{C^{1/z}}{z k!} \Gamma\left[k - \frac{1}{z}, C, CT^{z}\right] \propto k^{-\gamma}$$

- Degree distribution 2D

Assortative interc.

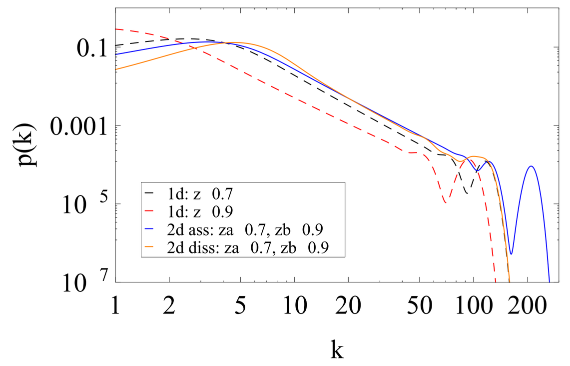
$$\varrho(t_a, t_b) = \frac{1}{T}\delta(t_a - t_b)$$

$$p(k) = \sum_{t=1}^{T} e^{-\bar{k}(t,t)} \times \bar{k}(t,t)^k \frac{1}{k!}$$

Disassortative interc.

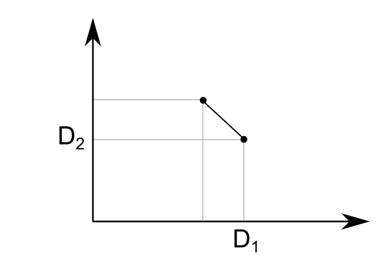
$$\varrho(t_a, t_b) = \frac{1}{T}\delta(t_a - (T+1-t_b))$$

$$p(k) = \sum_{t=1}^{T} e^{-\bar{k}(t,T+1-t)} \times \bar{k}(t, T+1-t)^k \frac{1}{k!}$$



Multiplex approach (outlook)

- Overlap measures topological similarity between different layers
- High overlap implies common multidimensional metric space
- Formulate parameter $0 < p < 1$ to characterize metric property



- $p \approx 0$: subspaces are different
- $p \approx 1$: subspaces form common metric space

Outlook: Investigation of multiplex metric properties.